# Laminar Separating Flow Over a Prolate Spheroid

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**Skin-friction lines and separated vortex sheets are visualized for the flow over a prolate spheroid at various angles of attack. Dyes and particles are employed in combination with laser sheet cuts. The data are digitized and the results of many realizations are collected on a composite figure. It is found that a region exists between primary and secondary separation that involves very large crossflows. It appears that for angles of attack of 6–30 deg, open separation is at hand.**

## I. Introduction

THE flow over aircraft fuselages and missiles may separate, generating large forces normal to their longitudinal axis. A variety of mechanisms can trigger steady or unsteady flow separation over such bodies. For example, separation may be induced by spikes, by the body nose, or by shock waves.[1] Equally important is the case of separation induced by nonzero incidence. In the latter case and for large angles of attack, separation may proceed in the form of alternate shedding.[2] For moderate angles of attack, the separated region is dominated by the axial flow and the wake is stable.[2] In the present paper, we study experimentally the flow over an axisymmetric body at incidence, with emphasis on the line of separation and the shape of the wake. This study is proposed as a bench-mark case for computations of laminar flows before the added complication of transition and turbulence are incorporated. Direct comparison can be facilitated because our flow visualizations have been digitized and are available on tape.

Numerical solutions of problems involving massive separation in two dimensions are very sensitive because small errors in the estimates of separation location lead to results that appear reasonable but may be far from reality. Three-dimensional separating-flow solutions have been considered recently. These too are very sensitive to separation location.

A good number of careful boundary-layer calculations have been carried out in the past decade for the flow about a prolate spheroid.[3–7] These calculations have clearly demonstrated that the difficulty of the problem centers mostly about the proper choice of the grid shape and marching process. Most recently, further improvements on methods of integration that conform with the principles of influence and dependence have appeared.[8,9] The difficulties multiply as one approaches the line of separation. All investigators encounter severe difficulties if they approach separation from the leeward side. Taking into account the errors involved in solving numerically the boundary-layer equations in the vicinity of separation and the fact that these equations are not even valid in this region, the usefulness of such methods becomes questionable. Moreover, their application to more general body shapes is a formidable exercise, requiring the constant attention of experienced investigators.

Approximate methods are available, such as integral methods[10] or streamline methods.[11–13] These methods are considerably simpler than the methods based on the complete three-dimensional boundary-layer equations, but the accuracy of their results is questionable, especially in cases of nonconventional body configurations. Full Navier–Stokes equations were presented most recently in Refs. 14–16. Once again, it appears that a fine grid is required in the vicinity of separation in order to model the phenomenon correctly.

Extensive and carefully obtained experimental data for high Reynolds-number flows over prolate spheroids have been presented by Meier and associates.[17–20] However, turbulent flow tends to smear out the detailed structure of separating flow. The present experimental evidence indicates that for laminar flows, the boundary layer develops exceptionally high crossflows near separation. Navier–Stokes solutions capture such phenomena and therefore should be very sensitive to the choice of the grid size in these regions. It is hence felt that a thorough investigation of the laminar flow structure, both experimental and numerical, is necessary before turbulence models are added to a Navier–Stokes code.

Historically, Maskell[21] was the first to identify the fact that separation in three dimensions could follow a distinct pattern that has no counterpart in two dimensions. He recognized that a separating vortex sheet could have a beginning and, thus, the oncoming stream could reach both its sides from upstream. He also identified a more conventional pattern whereby skin-friction lines emanating from stagnation could not reach behind the separating vortex sheet. He coined the terms "vortex separation" and "bubble separation" for the two patterns, and he accompanied his discussion by the sketches shown in Fig. 1. Numerical investigation of the topography of these patterns was initiated by Wang[5,22] who introduced the alternative terms "open" and "closed separation." Many other authors addressed the problem numerically[3,7–16]. Most recently, Costis and Telionis[13] pointed out another feature of three-dimensional separation that is not present in two dimensions. In three-dimensional flow, the vortex lines that represent the vorticity contained in the boundary layer cease to be parallel to the line of separation.

The experimental effort reported here is undertaken with the purpose of shedding some light on the peculiar structure of three-dimensional laminar separation. Our experiments were conducted in a water tunnel. In an earlier contribution,[23] we reported results from our laser-Doppler velocimetry (LDV) measurements over a prolate spheroid. The structure of the flow in the neighborhood of separation can be more clearly understood in terms of flow visualization.

A careful flow-visualization study of laminar flow over a prolate spheroid has been reported by Han and Patel.[24] In this

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study, the authors present detailed skin-friction topographies and discuss them in the light of Lighthill's theory (see Ref. 25 for extensive discussion). The visualizations in Ref. 24 were obtained by dyes released near the nose of the spheroid. The details of the flow in the immediate neighborhood of separation are not clearly captured. In the present paper, we provide visualizations obtained by dyes released in the neighborhood of separation. Moreover, we present information above the surface of the spheroid. This essentially consists of cross sections of the vortex sheets that were obtained by plane laser cuts. Our evidence indicates that only open separation is present for angles of attack of 6–30 deg.

## II. Experimental Arrangement

Experiments were conducted on a prolate spheroid model with axes ratio $b/a = 1/4$. The model was machined out of aluminum and was mounted by a strut from the one end, as shown in Fig. 2. This permitted adjustment of the angle of attack. The tests were conducted in a water tunnel at speeds corresponding to Reynolds numbers of the order of 10,000. Dye ports were drilled at strategically chosen points. Colored dyes were supplied by small flexible hoses that were fed and controlled by hypodermic needles. A novel method was also introduced in the present study. Thin hypodermic needles were mounted on the surface of the body, allowing the dyes to be released at the edge of the boundary layer. Still color photographs§ were taken from views along inclinations $\phi = 0$, 90, 120, and 180 deg (Fig. 3). These correspond to windward, side, oblique, and leeward views, respectively.

The visualization of shear layers in two dimensions is straightforward. (At high Reynolds number free shear layers are thin and often are referred to as "vortex sheets.") Dyes can be released from the wall in the attached boundary layers. The dyes follow the wall and eventually lift off at separation. Streaklines are thus generated emanating from the points of separation. Such dye lines can easily be photographed. In three dimensions the vortex sheets form nondevelopable surfaces. Uniformly marking such sheets with dye would obscure the visualization process. It was thus decided to obtain cuts of the vortex sheets. To this end, the attached boundary layers were seeded with silicon carbide particles. Such particles are manufactured in sizes of a few microns, track the flow accurately, and have excellent scattering characteristics. They are usually employed for seeding in laser-Doppler velocimetry. In the present case, seeding was confined to the boundary layers and, therefore, the free shear layers. It was then possible to visualize cross sections of the shear layers by employing laser sheets normal to the axis of the model.

## III. Skin-Friction Line Visualization

Experiments were conducted for angles of attack $\alpha = 3$, 6, 10, 15, 20, and 30 deg. The results confirmed the fact that for very low and very high angles of attack, here for $\alpha = 3$ deg and for $\alpha > 30$ deg, the line of separation is closed around the body. Two critical points can then be identified on the line of separation and the general pattern fits nicely with one of the models described by Maskell[21] (see discussion in Refs. 3 and 23), namely, the bubble model. This, in the more common terminology, is the case of closed separation. Open separation is more interesting in practice, and is essentially the topic of the following discussion.

The coordinate system employed is defined in Fig. 3. The azimuthal angle $\phi$ is measured from the windward side of the model. In Figs. 4–7, we display flow visualizations of different views of the flow about a prolate spheroid at angles of attack of 3, 6, 10, and 20 deg, respectively.

§A collection of color photographs is available upon request. Only black and white photographs are displayed in this paper.

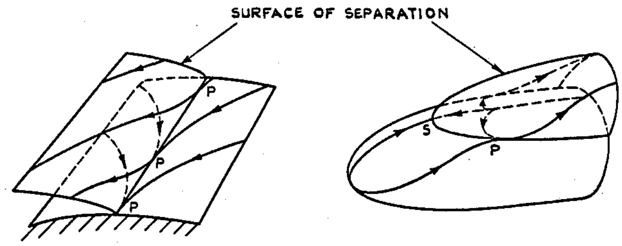


Fig. 1 Maskell's sketches of vortex and bubble separation.

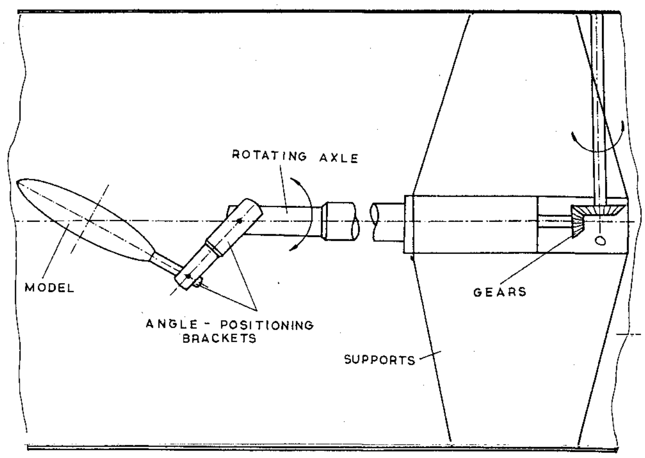


Fig. 2 Model and support.

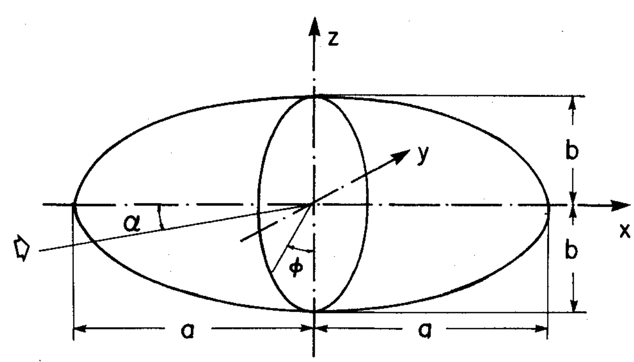


Fig. 3 Definition of coordinates.

In Fig. 4, a well-behaved closed-separation pattern is identified in agreement with all analytical predictions. While this paper was under review, Cebeci[26] communicated with the authors his numerical results, which indicated that even for very small angles of attack separation is of the open type. The main concern with our experimental work was whether the mounting sting interferes with the flow. To minimize this effect, we machined a new sting with a radius equal to $b/6$. Moreover, we provided the model with dye ports closer to the separation region. A large number of tests were repeated and the results will be included in a new report. A representative visualization is shown in Fig. 4. It appears that for the conditions of the experiment, separation is closed. However, the authors believe that the only way to make a convincing argument is to repeat calculations for a body in the shape of our model, including the sting.

Details of the flow, uncovered for the first time here, shed light on controversial interpretations of earlier numerical results. For angles of attack 6 deg $\leq \alpha \leq$ 30 deg, the evidence clearly indicates that there are two lines of open separation. The dyes remain attached to the body and lift off along two

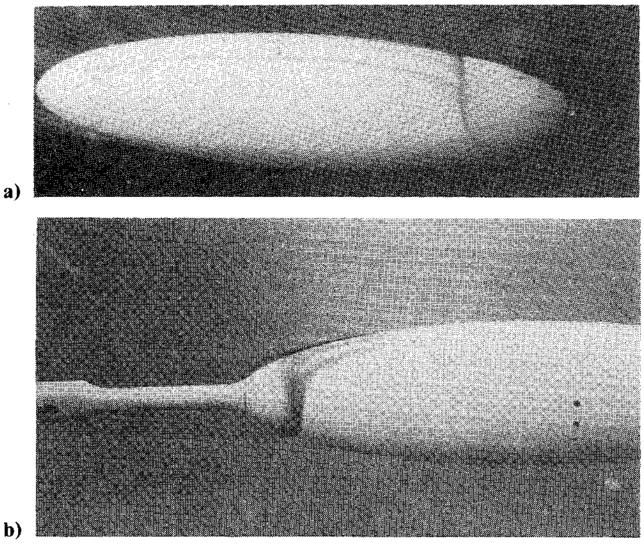

Fig. 4 Skin-friction lines for $\alpha = 3$ deg: a) side view ($\phi = 90$ deg); b) view from $\phi = 120$ deg.

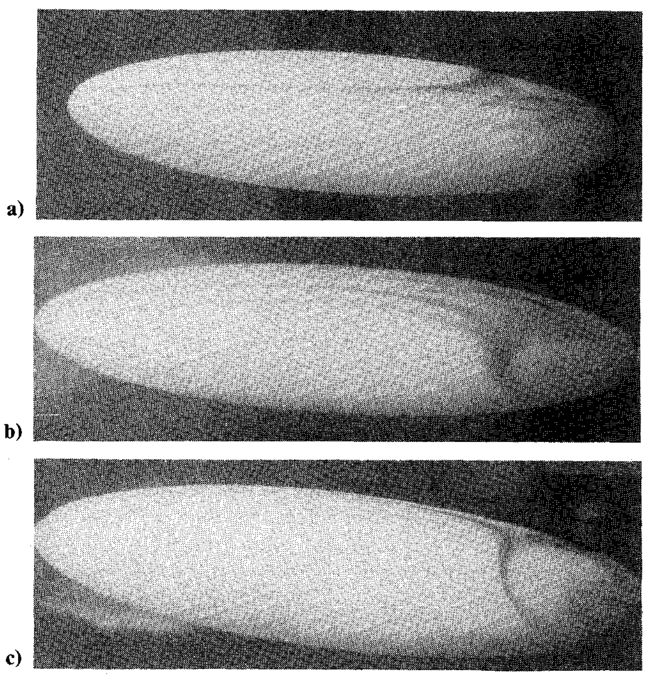

Fig. 5 Skin-friction lines for $\alpha = 6$ deg: a) leeward view; b) view from $\phi = 120$ deg, and c) side view.

open separation lines. This observation was also made earlier by Han and Patel.[24] For this range of angles of attack, it appears that the basic skin-friction line pattern is qualitatively the same as described in the following paragraph.

Three distinct regions difficult to be accessed from upstream become quickly apparent and may be erroneously interpreted as regions of bubble separation. The first region extends over a portion of the leeward side and appears dark in the top view of Fig. 6. It should be emphasized that all of the darkened area in this figure is due to dye that has been emitted at only one port in the front part of the model. In the same figure, the two lighted areas on the two sides in the aft of the body are strongly suggesting the existence of two separated regions of the closed type. The pattern could be sketched approximately as in Fig. 8. What has been known for a long time is that, in fact, the flow remains attached on the leeward plane of symmetry and in its neighborhood. This is corroborated by the fact that the dye stays on the wall until it reaches the small

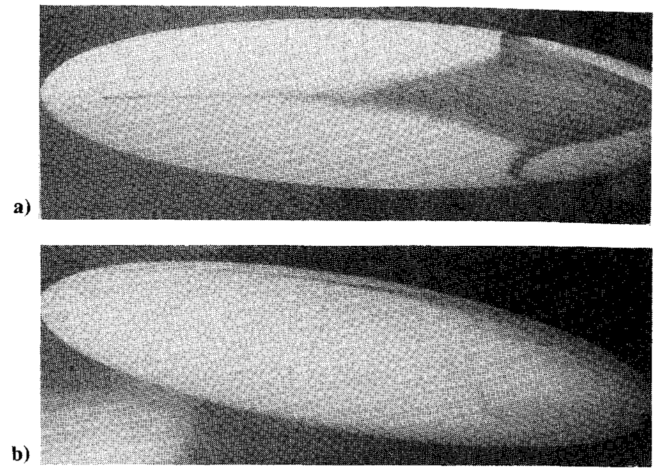

Fig. 6 Skin-friction and inviscid lines for $\alpha = 10$ deg: a) leeward view, and b) view from $\phi = 120$ deg.

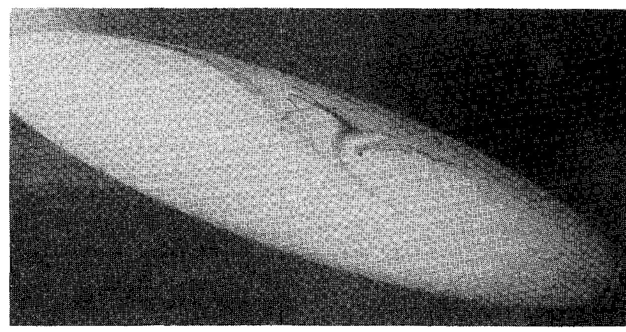

Fig. 7 Skin-friction and inviscid lines for $\alpha = 20$ deg. View from $\phi = 120$ deg.

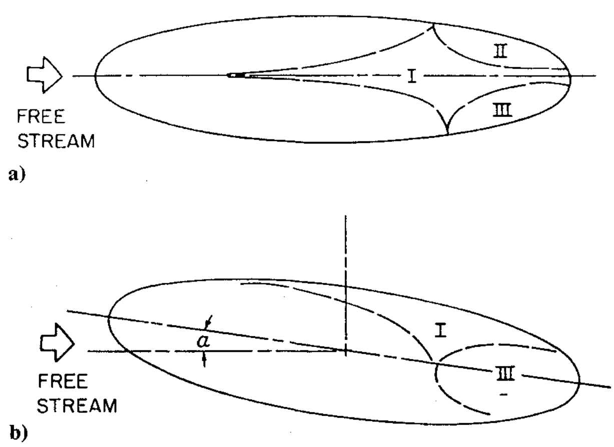


Fig. 8 The three regions of difficult accessibility for $\alpha = 10$ deg: a) leeward view, and b) view from $\phi = 120$ deg.

mount at the aft of the body. However, our evidence indicates that what appears as closed separations (bubble type) on the sides of the model are actually regions of incredibly large crossflows. This is shown more clearly in the flow visualization of Fig. 9. In this figure, the inviscid streamlines as well are marked by dyes released at the edge of the boundary layer.

## IV. Laser Cuts

It was found during the skin-friction visualization experiments that dye injected from a point located on the upper meridional plane and in the front part of the body can reach almost any point of the vortex sheet immediately above the

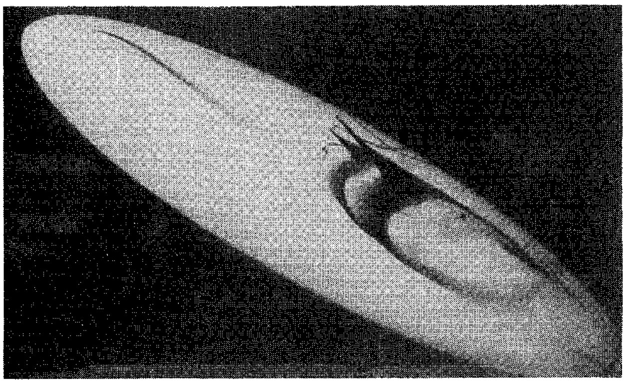

**Fig. 9 Skin-friction and inviscid lines for $\alpha = 30$ deg. View from $\phi = 120$ deg.**

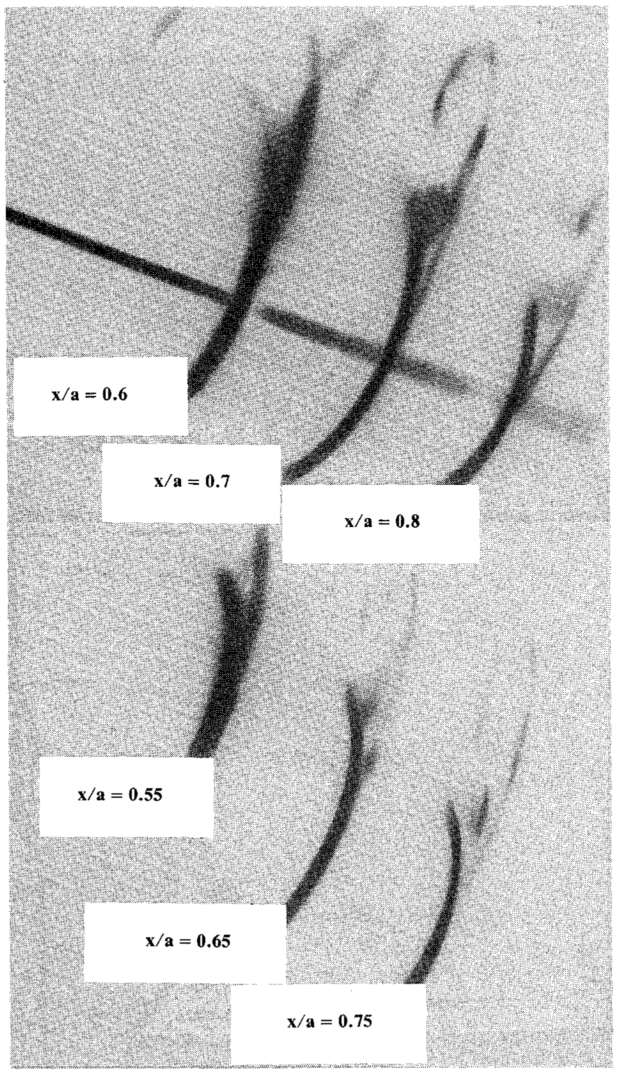


**Fig. 10 Laser cuts of separating vortex sheets at different axial locations for a spheroid at $\alpha = 30$ deg.**

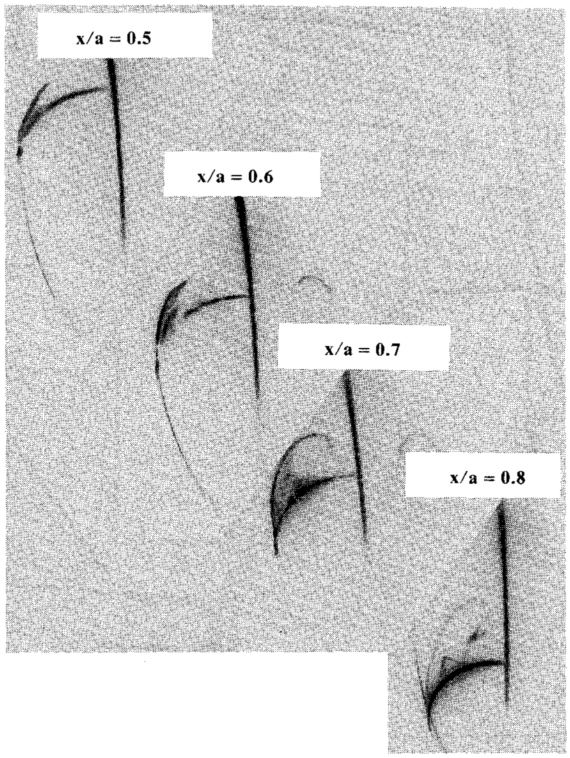


**Fig. 11 View from downstream of the field of Fig. 9.**

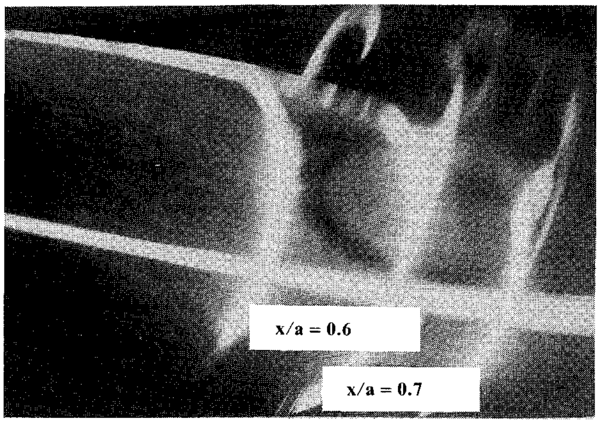


**Fig. 12 Composite flow visualization of skin-friction lines with vortex sheet cuts.**

line of separation. The fact that dye injected in the flow from one point is dispersed in such a way that an entire region can be visualized is perhaps a feature of the external flow about the prolate spheroid. The technique that was developed employs particle seeding and planes of laser light. Cross sections of vortex sheets were thus generated.

A mixture of silicon carbide particles and water was injected from a port located in the upper meridional plane and in the front part of the body. A laser beam was expanded into parallel planes of light by means of cylindrical lenses. The planes of light were positioned at various axial locations, perpendicular to the model axis. The particles in the lighted planes scatter the light and thus two-dimensional cross sections of the separated vortex sheets can be clearly visualized. With the plane of light directed normally to the main axis of the spheroid, the two-dimensional cross sections can be used to obtain quantitative results.

Releasing dyes in two- and three-dimensional steady flows allows us to visualize streamlines or stream surfaces, respectively. However, if the dyes or seeding particles are released on the surface of the body, and if the corresponding coefficient of species diffusion is not much different from the coefficient of molecular diffusion, then the dyes diffuse and convect together with vorticity. This is of paramount importance in the present study, because it implies that the cross sections of the sheets visualized in space are not just stream sheets but free shear layers or equivalently vortex sheets.

Pictures of cross sections of the separated vortex sheets using this method are presented in Fig. 10 for an angle of

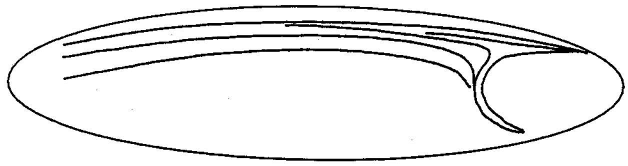

Fig. 13 Side view of digitized skin-friction lines for an angle of attack $\alpha = 6$ deg.

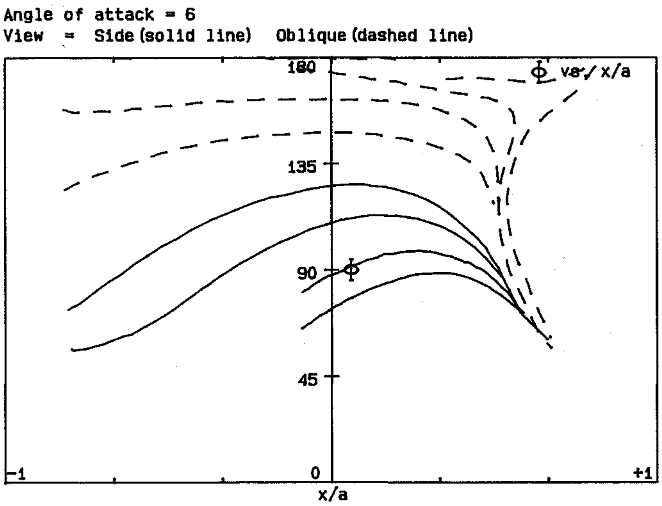


Fig. 14 Digital data for skin-friction lines for $\alpha = 6$ deg in an $x - \phi$ plane, obtained from: ---, $\phi = 180$ deg, —, $\phi = 90$ deg.

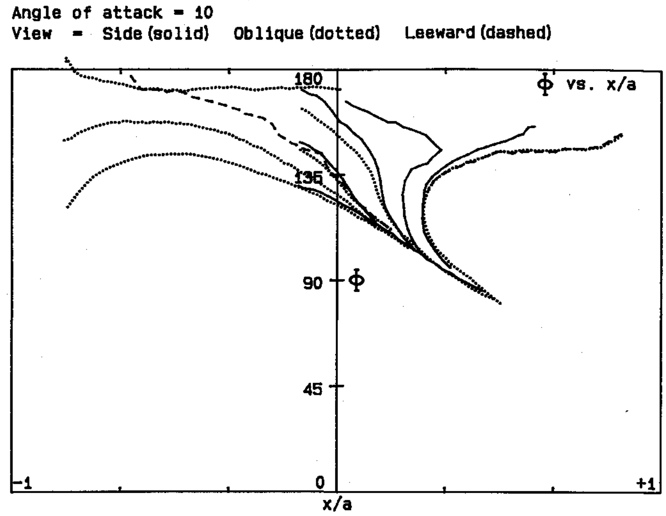


Fig. 15 Composite figure of digital data of skin-friction lines for $\alpha = 6$ deg in an $x$–$\phi$ plane, obtained from:---, $\phi = 180$ deg,. . . , $\phi = 120$ deg; ——, $\phi = 90$ deg.

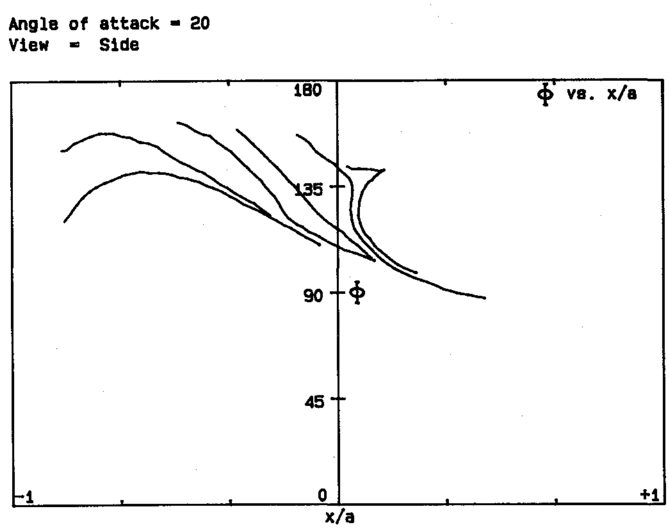


Fig. 16 Digital data of skin-friction lines for $\alpha = 20$ deg in the $x$–$\phi$ plane, obtained from $\phi = 90$ deg.

attack $\alpha = 30$ deg and $Re = 11{,}700$. It should be noted that in each frame of this figure, three cuts are simultaneously visualized. Laser cuts of the vortex sheets as viewed from directly downstream are displayed in Fig. 11. The results of an effort to visualize simultaneously skin friction and vortex sheet information is shown in Fig. 12. This is the most important evidence that the vortex sheets separating along the primary and secondary separation merge within the thickness of the boundary layer to a single sheet.

As discussed earlier, the skin-friction line patterns appear to indicate the presence of two separated regions on the sides of the model. These were denoted as regions II and III in Fig. 8. Using the laser-cut visualization technique (Fig. 11), it was found that the upper and lower boundaries of those regions are indeed separation lines. Two free shear layers emanate from those separation lines in a direction almost tangent to the surface of the body. Figure 10 shows clearly the two shear layers. However, only in the lower boundary of these regions is the separation line formed in the accepted way: that is, by means of converging streamlines from both sides of the line. The separation line of the upper boundary of these regions reveals an entirely new topography of skin friction and separation lines on a three-dimensional body. Skin-friction lines converge to the upper boundary of such a region moving in the direction of the outer flow. However, the skin-friction lines inside regions II and III move away from the upper boundary and in the opposite direction to the freestream. The upper boundary of such a region, which is a line of secondary separation, is also a line of very strong reverse flow, almost approaching a boundary-layer crossflow of 300 deg.

Proof that such a region is not a closed-separation region is provided by our flow visualizations. The skin-friction lines, especially the one shown in Fig. 9, indicate that both sides of primary separation can be reached from upstream. Moreover, all laser-cut visualizations show that a little above the body surface there is only one vortex sheet developing on each side of the model. Actually, two vortex sheets are generated along the primary and the secondary lines of separation but they merge into one at a height above the surface of the body comparable to the boundary-layer thickness.

## V. Digital Processing

A very large number of flow visualizations, not displayed here, were digitized. The procedure was based on the assumption that a photograph represents a projection of all points on the spheroid along a plane normal to the azimuthal direction of view. Discrete coordinates $x,\phi$ were then calculated and stored.

The technique offers quantitative information that is stored on tape and can be used in direct comparison with analytical results. It is thus possible to transfer on a single figure information obtained by different realizations of flow visualization. It also becomes possible to unwrap the surface into a coordinate plane $x,\phi$ and display skin-friction lines from $\phi = 0$ to 180 deg on a single figure. Results obtained in this way are shown in Figs. 13–16. In Fig. 13, we display data as obtained from flow visualization. Figures 14–16 are composites of data obtained by different realizations and different views of the skin-friction lines. Such data, therefore, represent essentially an ensemble average of optical information.

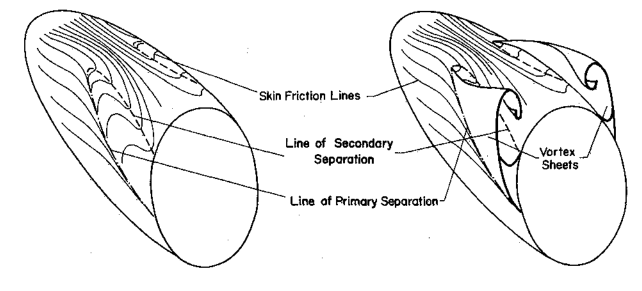


**Fig. 17 Schematic representation of the skin-friction lines and vortex sheets as obtained by flow visualization.**

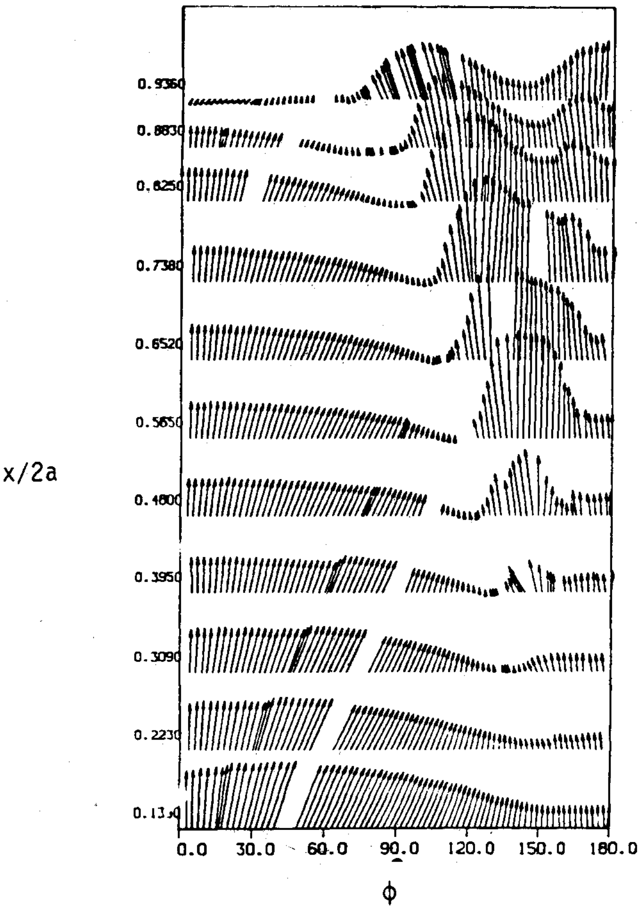


**Fig. 18 Shear stresses measured on a 6/1 spheroid at $Re = 1.6 \times 10^6$ (from Ref. 17).**

## VI. Discussion

The data presented here indicate that two vortex sheets are shed over a prolate spheroid for angles of attack $6 \text{ deg} \leq \alpha \leq 30$ deg. Each sheet appears to have a leading edge in agreement with Maskell's description of vortex-type separation or Wang's open separation. However, the foot of each sheet is rooted into two lines of separation. The primary line of separation is on the windward side.

The space between the primary and secondary lines of separation is the space that was identified as separation region II or III in the earlier discussion. In this space, the outer flow is still very nearly parallel to the wall. It is conceivable, therefore, that a boundary-layer calculation could be marched through it and reach the primary line of separation from the leeward side. However, it was found here that the skin-friction lines in this space are turned in a direction almost opposite to the direction of the outer flow. A schematic representation of the qualitative aspects of the flow is presented in Fig. 17.

The findings of the present paper are basically in agreement with the results and the interpretation of Han and Patel,[24] except that our data do not support the existence of a nodal point at the initiation of the separation line (poing N in the figures of Ref. 24). For higher angles of attack, Han and Patel propose a pattern of skin-friction lines, in the predominant direction of the outer flow (see, for example, Fig. 13 of Ref. 24). The present data appear to contradict this conjecture.

Earlier findings[17–20] indicate that at higher Reynolds numbers, there is a region on the leeside of separation where the flow turns suddenly turbulent. In Fig. 18, we display skin-friction distributions measured[20] on a prolate spheroid with an axis ratio $a/b = 6/1$ at a Reynolds number of $Re = 1.6 \times 10^6$. In the region that corresponds to our region between the primary and secondary lines of separation, the flow actually goes turbulent, as evidenced by much larger values of wall shear. The present study indicates that the velocity profiles in this region are inflectional and include large components in the crossflow direction. Apparently, then, the flow quickly goes turbulent and smears out the structure of the skin-friction lines depicted in Fig. 17. Therefore, for higher Reynolds-number flow, the flow could be laminar upstream but turn turbulent in the regions II and III of Fig. 8. The boundary-layer flow is, thus, well mixed and conforms with the direction of the outer flow. A more conventional open-separation pattern is then recovered, whereby skin-friction lines merge smoothly on both sides of the only line of separation.

The skin-friction lines within the regions II and III follow an S-shaped pattern. In view of this evidence, we feel that Cebeci et al.[8] simply reached or rather came very close to the point of zero axial shear stress when they were forced to abandon their integration on the leeside of the body. This possibility was already identified in their presentation. Apparently, the locus of such points may be impossible to cross with conventional numerical schemes or, in their terminology, such flows are "incalculable." However, more recent calculations employing full Navier–Stokes equations[14–16] clearly capture the structure of S-shaped lines presented here.

Smith[27] argues that for an infinite Reynolds number the angle of separation must be zero. This angle is usually defined as the angle formed by the vortex sheet and the surface of the body. The argument of Ref. 27, paraphrased by the present authors, goes as follows: If the separating vortex sheet meets the wall at an angle other than zero, then for $Re \to \infty$, i.e., infinitely thin boundary layers, the velocity at the point of separation is zero. Hence, vorticity, which is convected with the boundary-layer flow, would be trapped there and could not be shed in the separating vortex sheet. However, in both two- and three-dimensional flows, as indicated here, there is always secondary separation as well. The present results indicate that the two sheets emanating along the lines of separation (see Fig. 17) coalesce and together lift off the wall at an angle that is not equal to zero. Along the secondary separation vorticity of the opposite sense may be released in the wake and this would reduce the overall strength of the vortex sheet.

The basic pattern of skin-friction lines is consistent with the solution and the description of three-dimensional separation proposed for the first time by Sears.[28]

## VII. Conclusions

The present experimental data indicate that open separation develops in the form of two vortex sheets emanating along a primary and secondary separation line, respectively. These sheets quickly combine to form a single sheet. The space between the two separation lines contains regions of very high values of crossflow. It is, therefore, suggested that numerical methods may encounter difficulties in this region. Boundary-layer calculations have indeed proven that it is very difficult to penetrate it.

The two lines of separation merge upstream and, if visualized by surface dyes, have the appearance of a bubble or closed type of separation. However, the two vortex sheets quickly merge into one that then lifts off and rolls in the conventional way. We, therefore, propose here a broader definition of the concept of open separation. We should define open separation as the flow whereby boundary-layer streamlines coming from the stagnation region can reach both sides of the vortex sheet as they lift off the surface of the body.

The flow structure in the leading edge of the two lines of separation is not yet well understood. It appears that the two separation lines turn and merge into each other. However, the detailed pattern of skin-friction lines in this neighborhood is not very clear.

## Acknowledgment

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